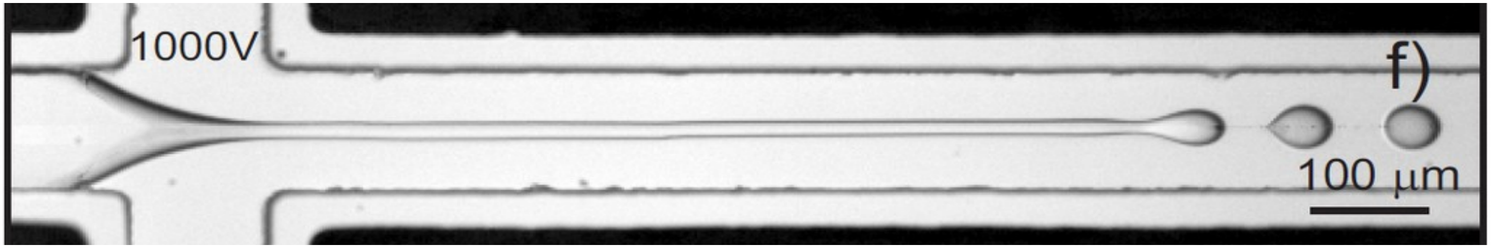
Long jets generated by ac electric fields in microfluidics

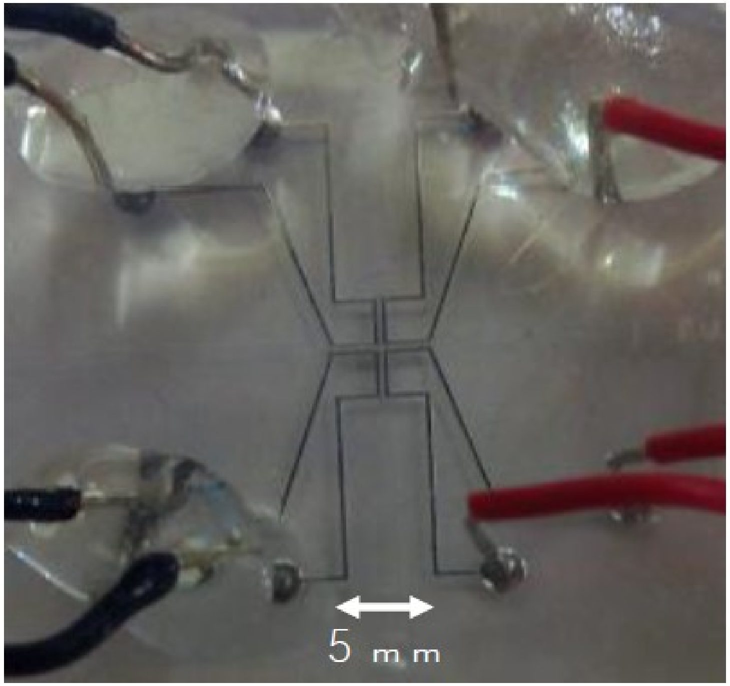


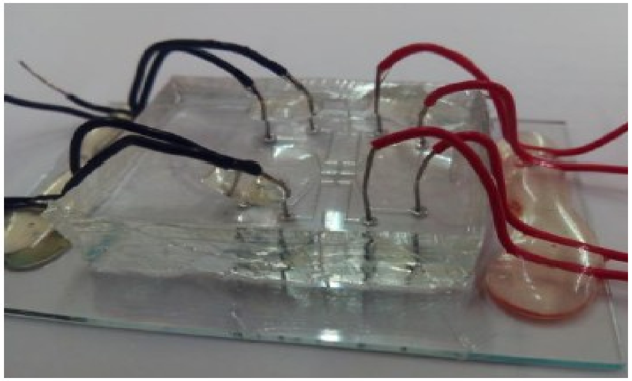
Pablo García-Sánchez
Elena Castro-Hernández
Antonio Ramos

IWEHD 2016
Poitiers, September 1-2

UNIVERSIDAD DE SEVILLA

A flow focusing device with electrodes along the channels





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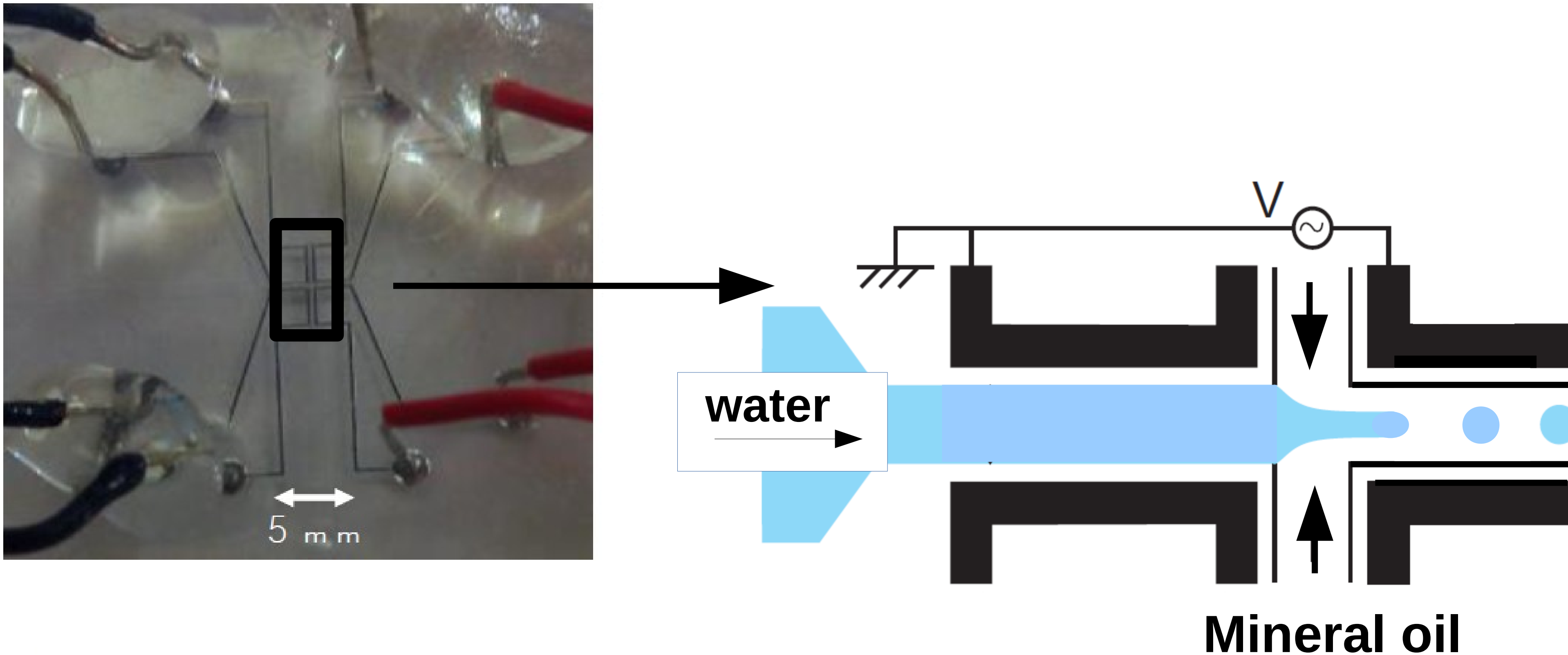

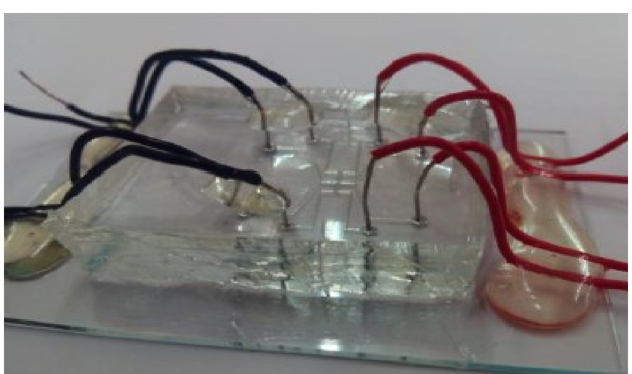

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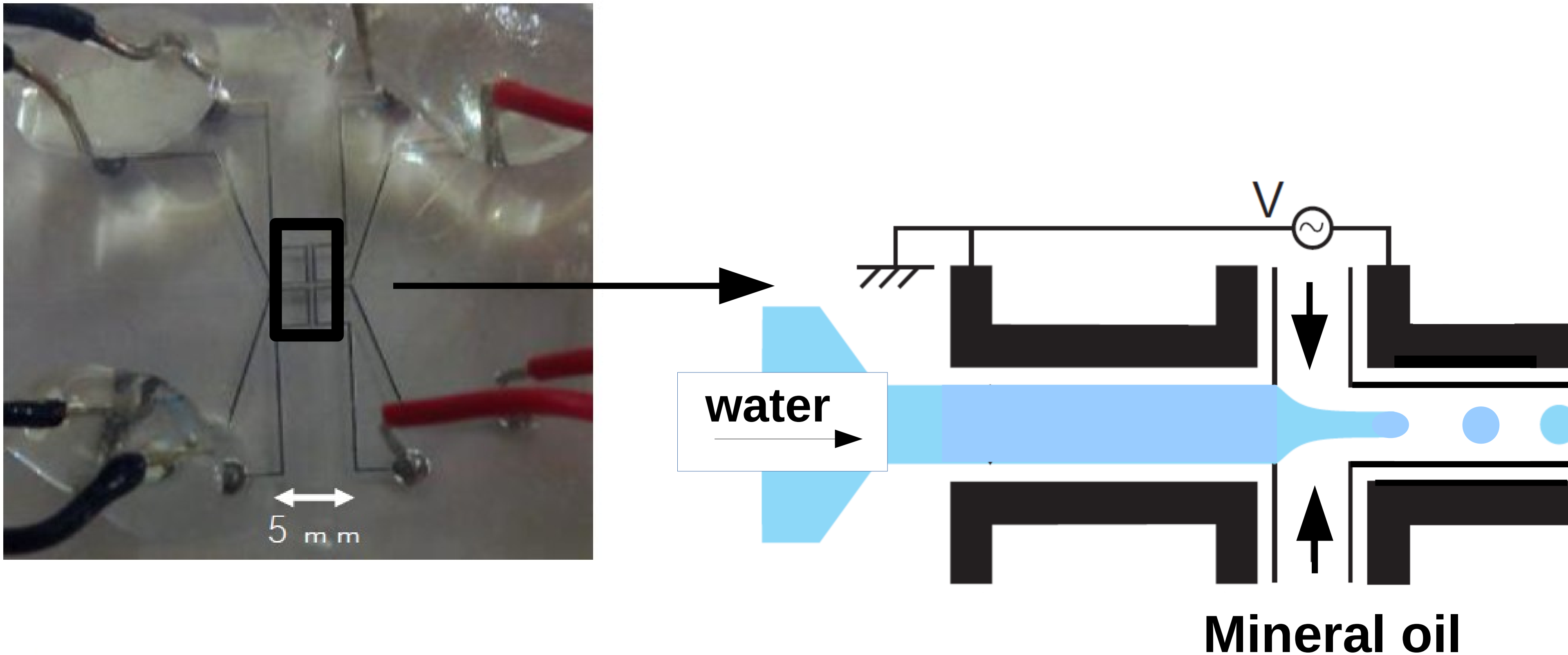
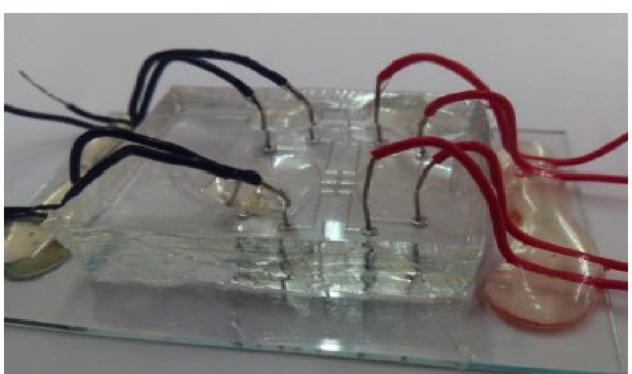
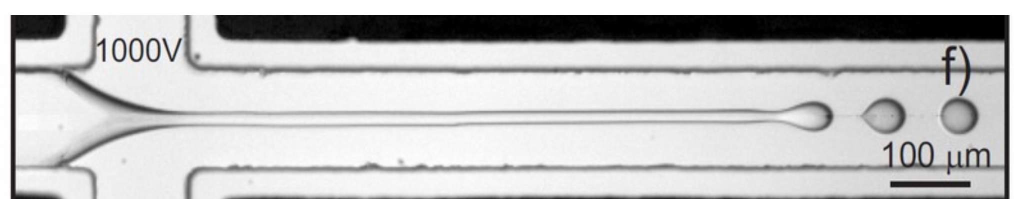
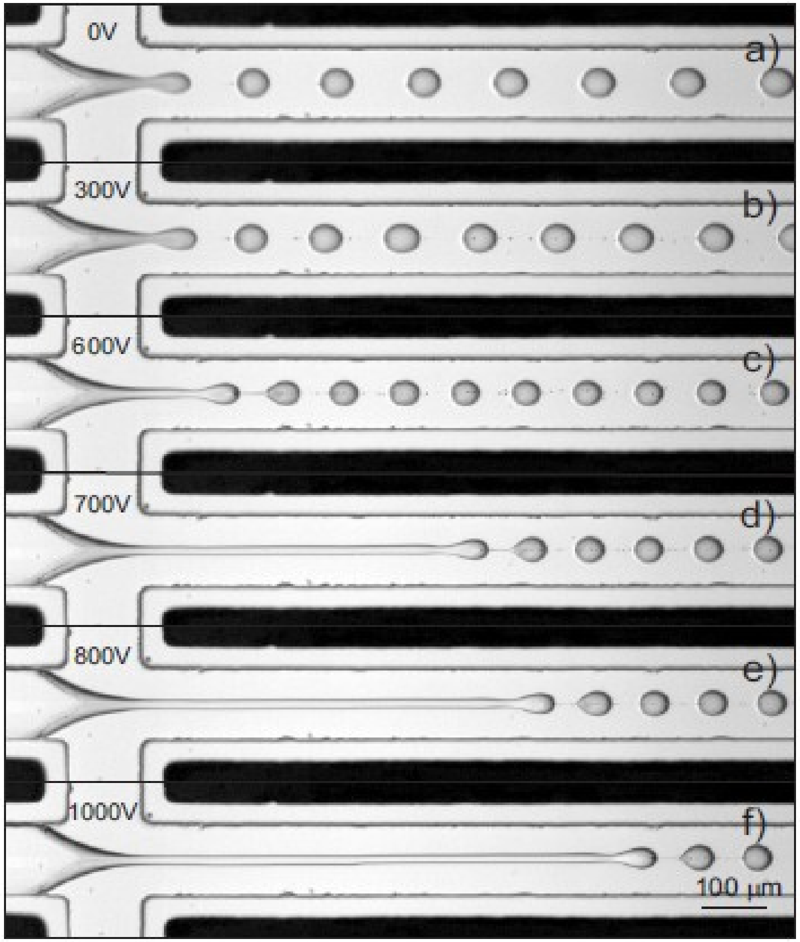




High Voltage!



Slender jets appear at high voltages



KCl 3 mS/m ; 9 kHz
Inner flow rate: 50 µl/h
Outer flow rate: 400 µl/h

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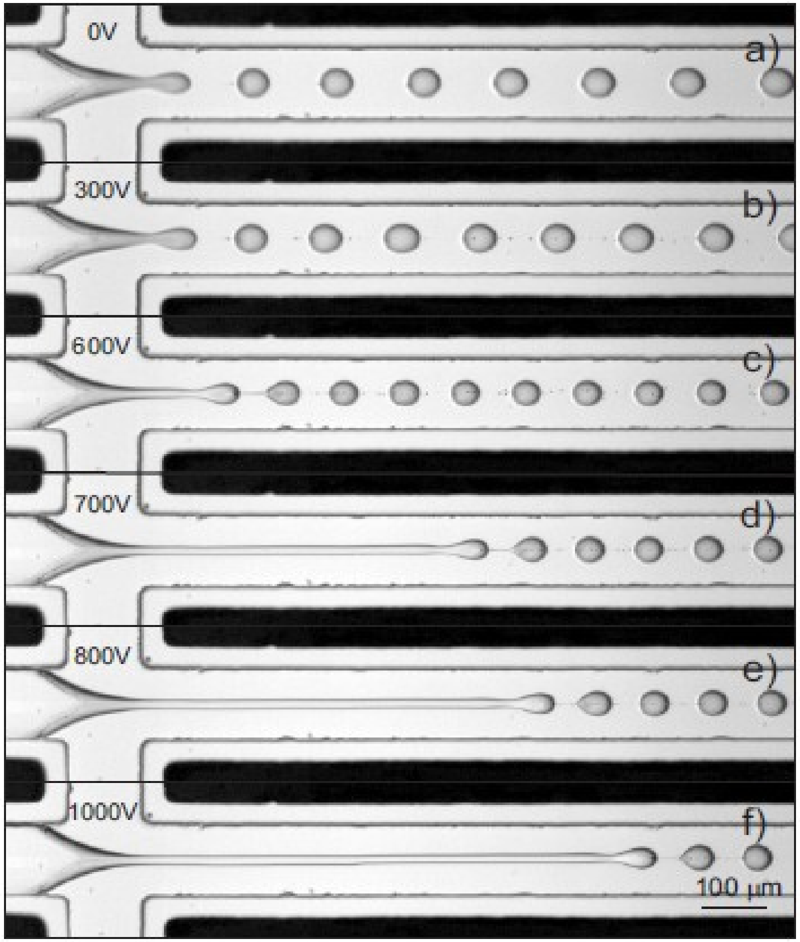

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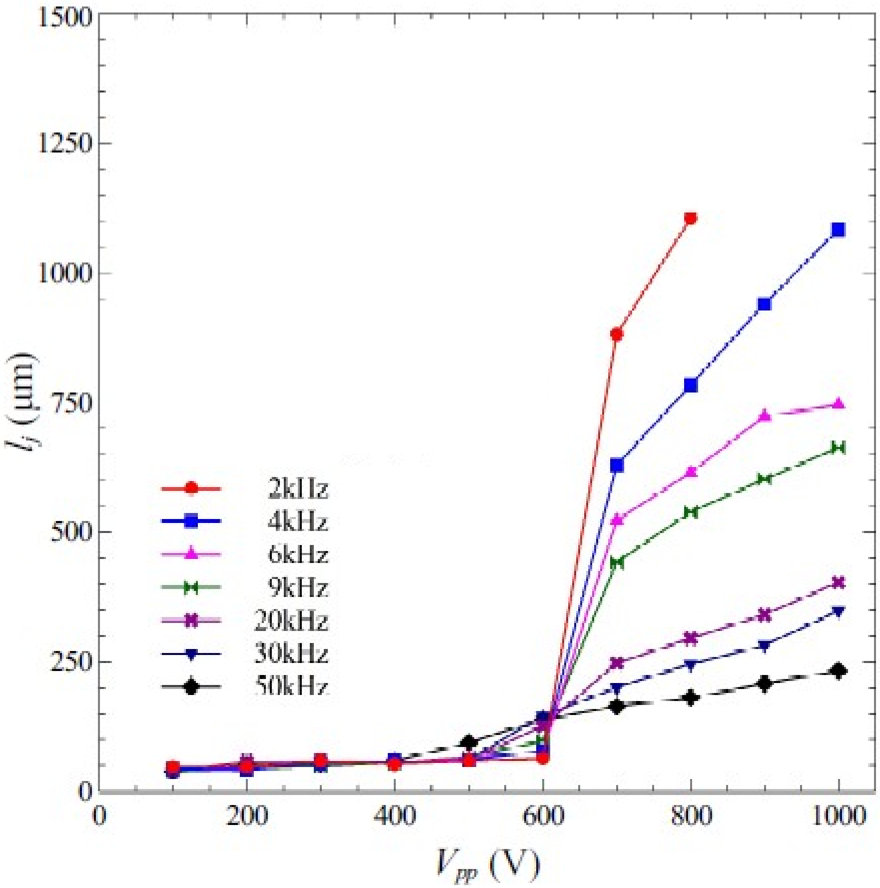
...and jet length depends strongly on frequency of the signal.



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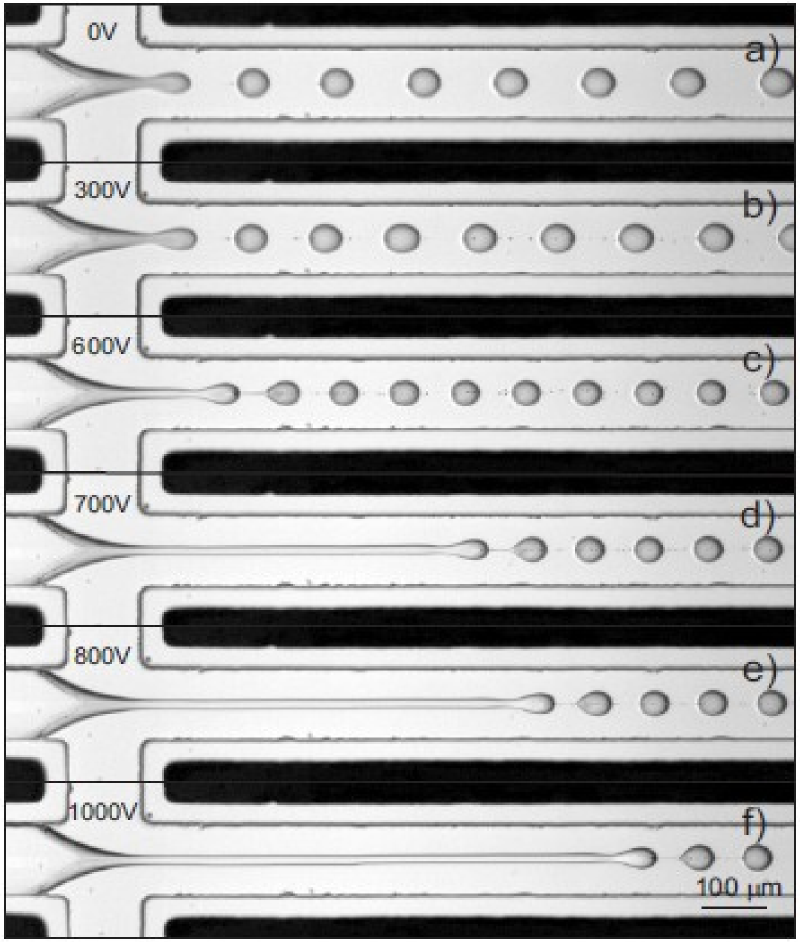

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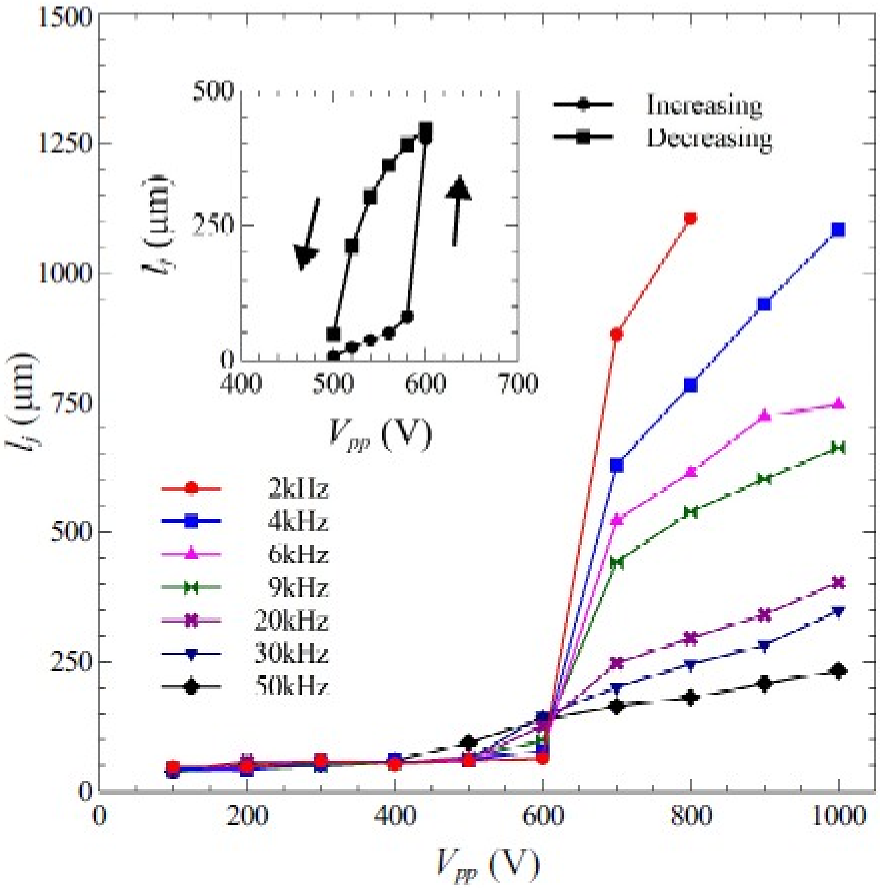

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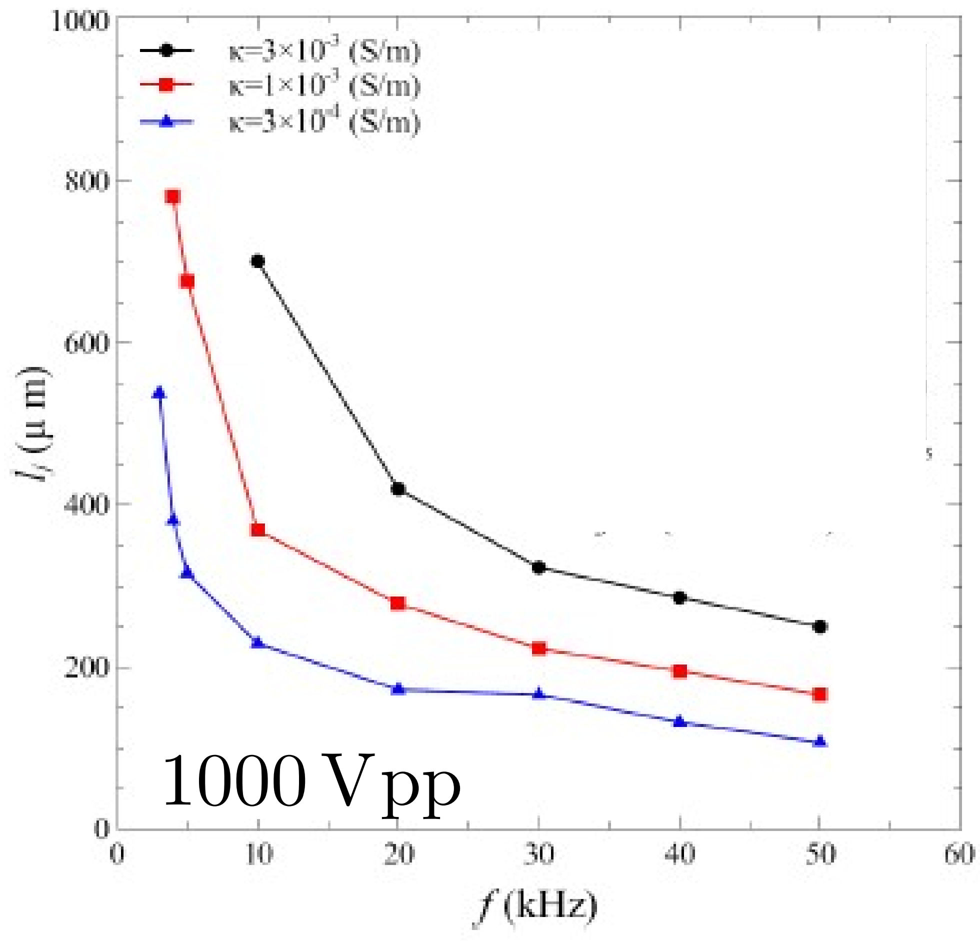

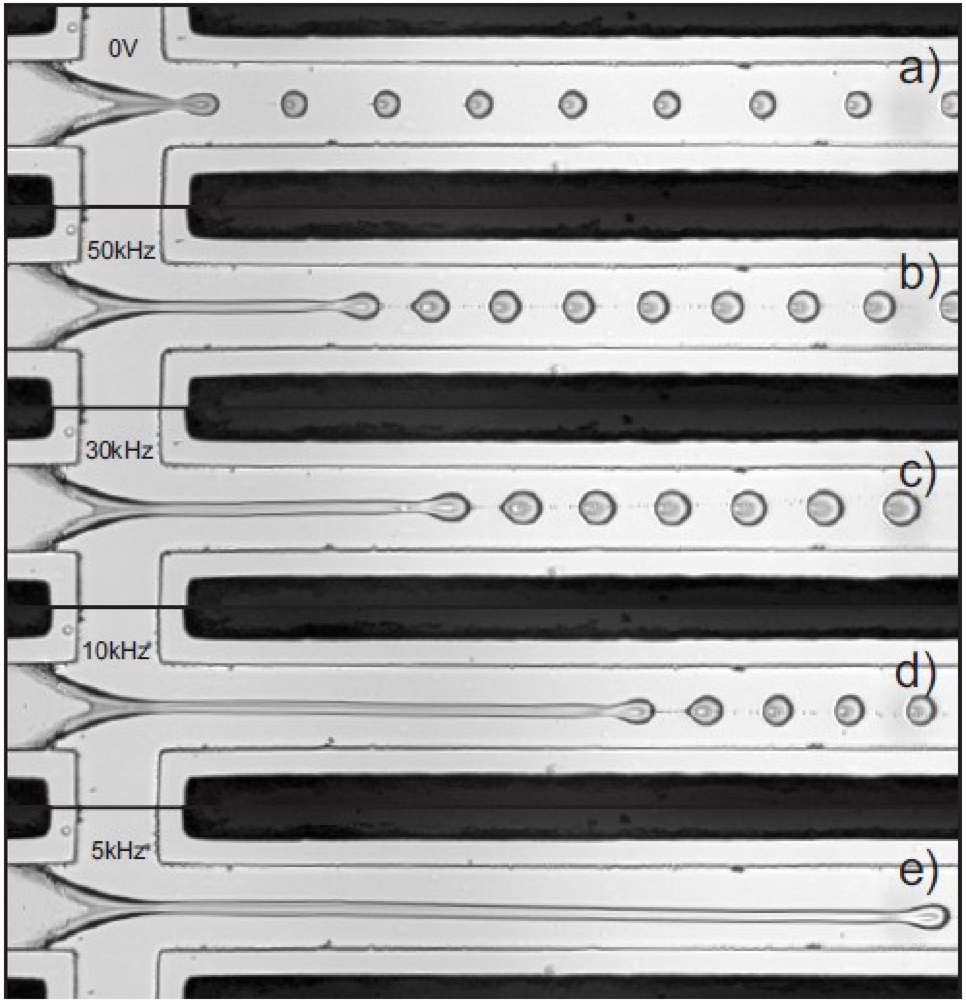

KCl 3 mS/m ; 1000 Vpp
Inner flow rate: 50 μl/h
Outer flow rate: 400 μl/h

Slender jets appear at high voltages

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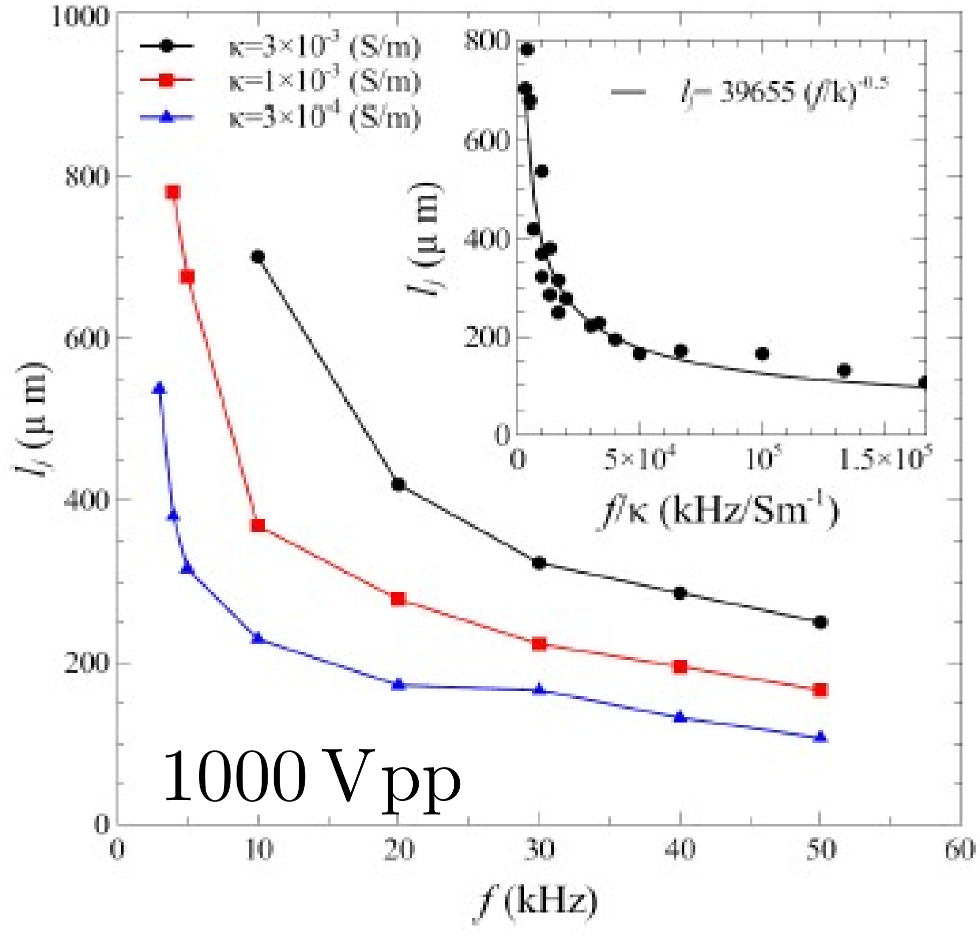

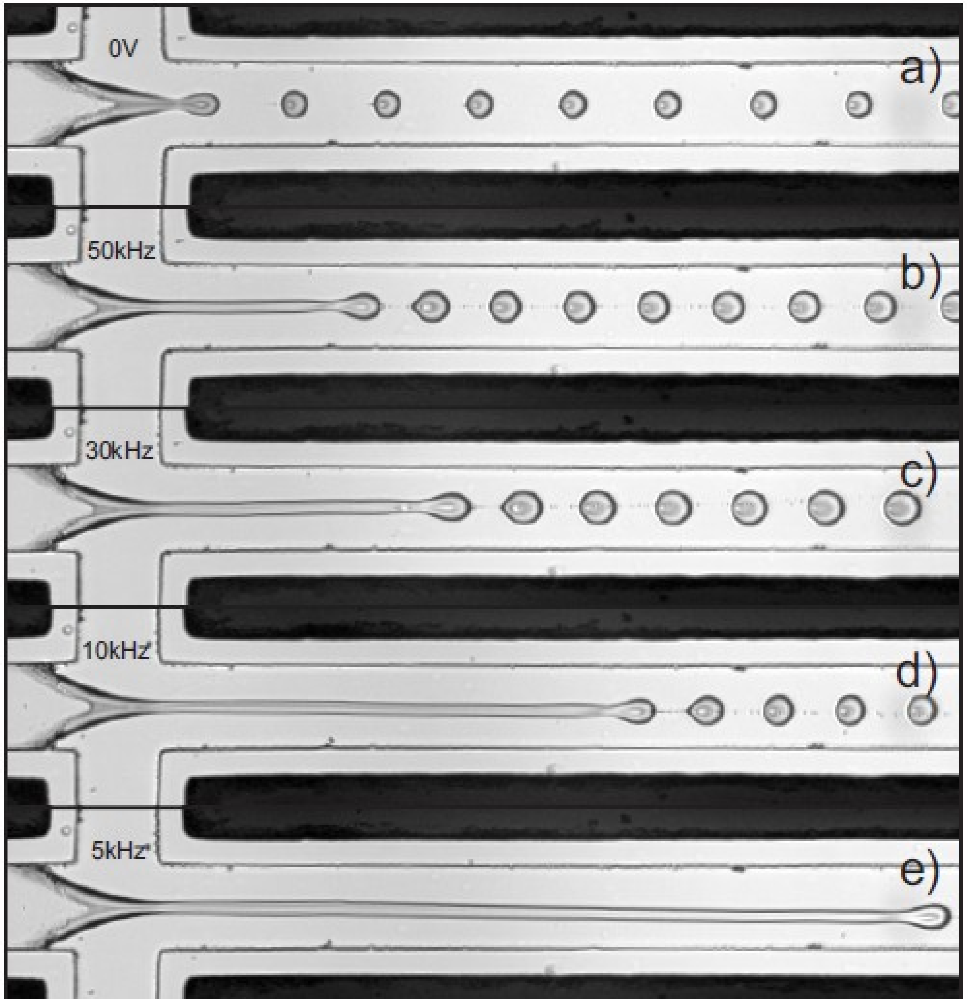

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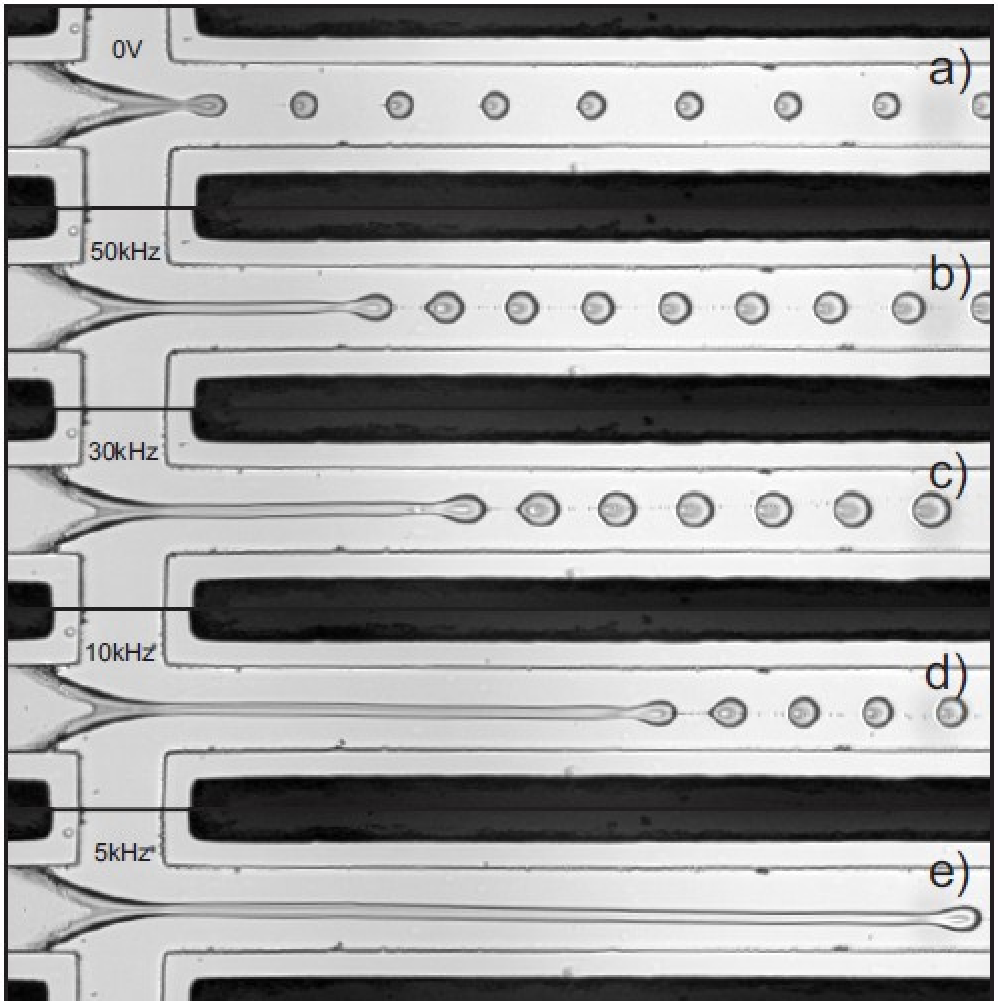
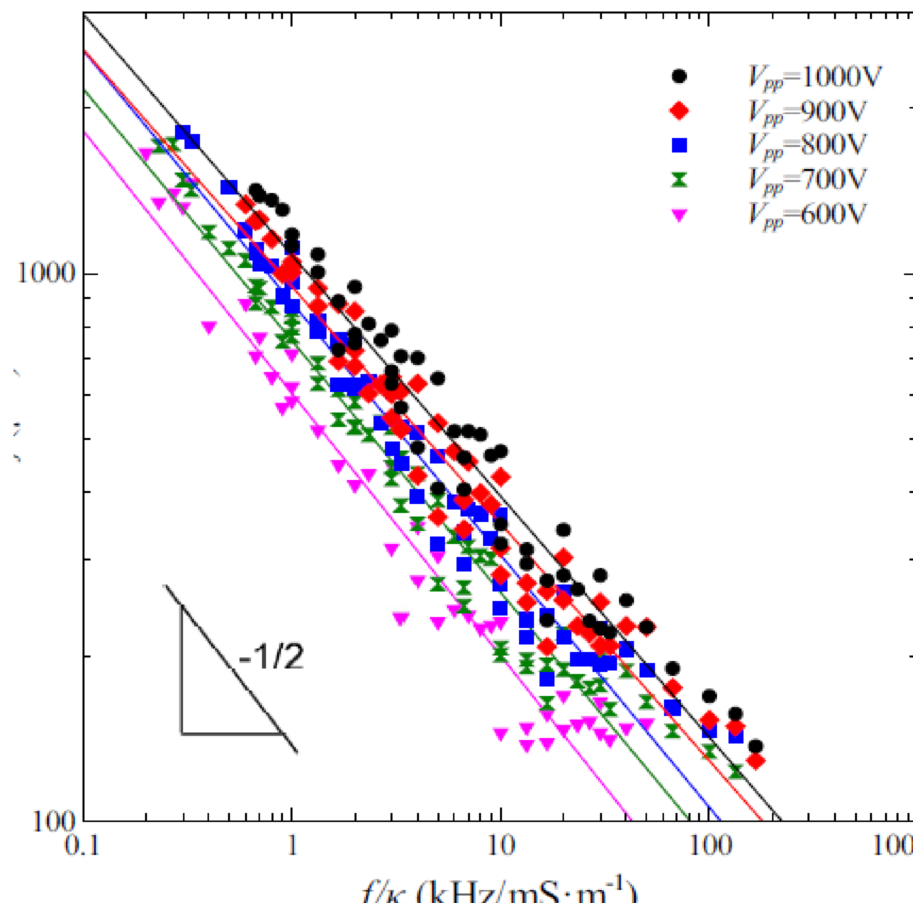


KCl 3 mS/m ; 1000 Vpp
Inner flow rate: 50 μl/h
Outer flow rate: 400 μl/h

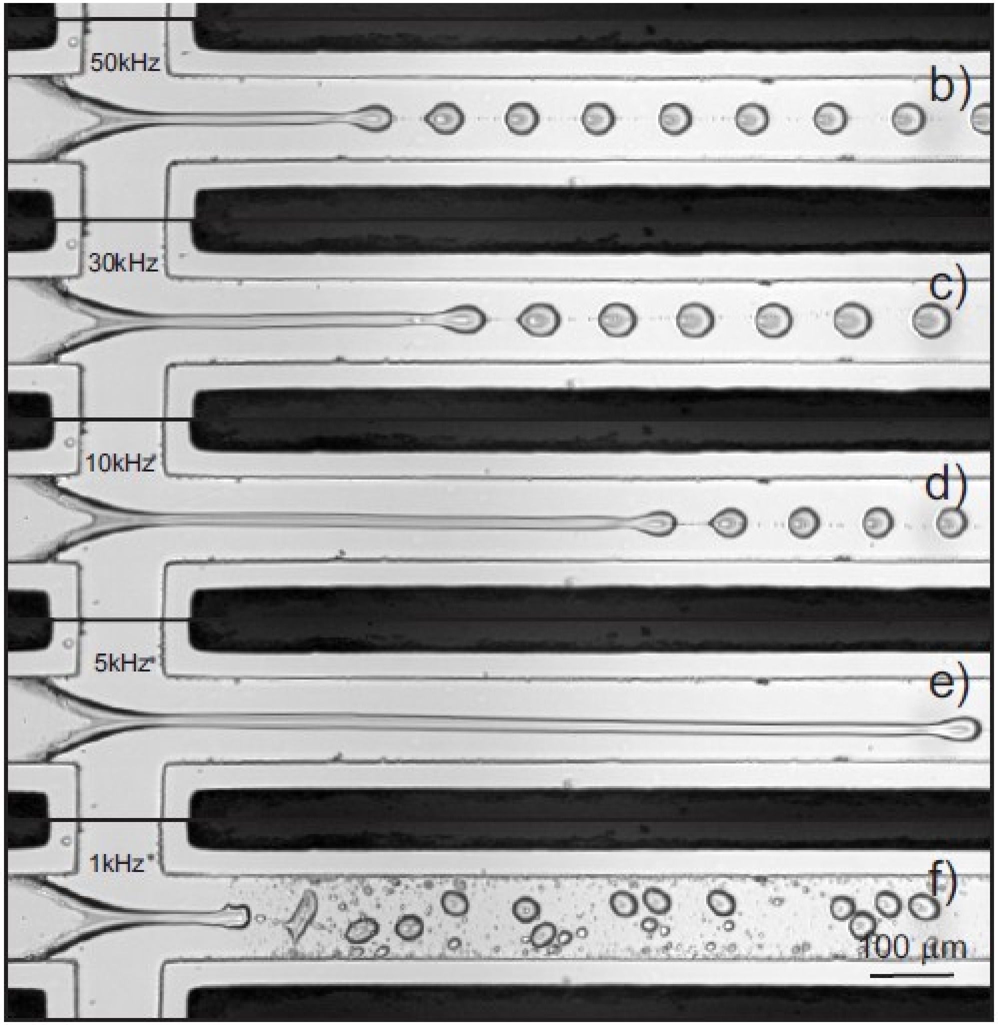
Jet length scales with $\sqrt{f/\kappa}$



..and for low frequencies the filament breaks into many small droplets

50 kHz

1 kHz



- Why do long jets appear at high voltages?

- What is the electric field along the jet?

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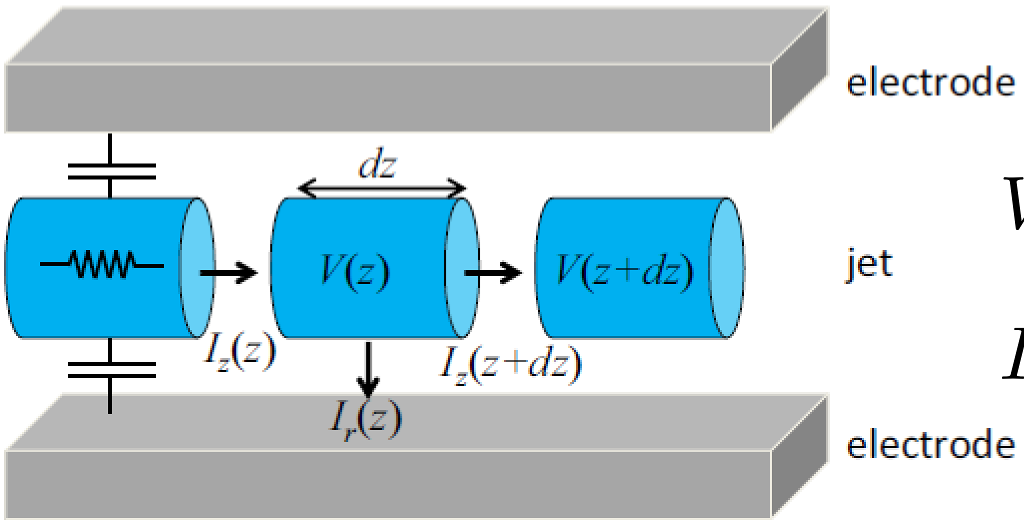
Distributed element circuit model



$$V(z) - V(z + \mathrm{d}z) = \frac{\mathrm{d}z}{\kappa\pi a^2} I(z)$$

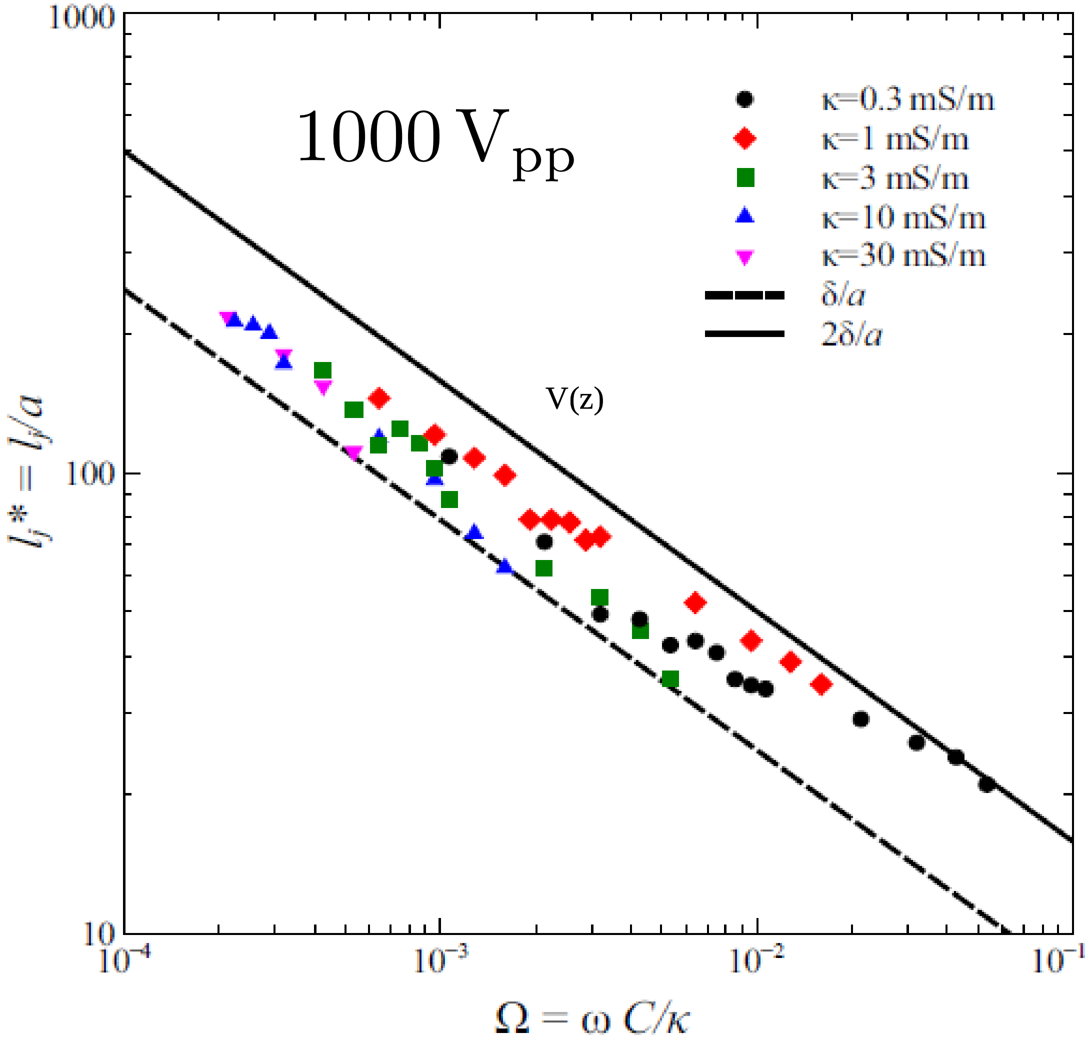
$$I(z) = I(z + \mathrm{d}z) + i\omega C V(z)\mathrm{d}z$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{i\omega C}{\kappa\pi a^2} V$$

Typical length

$$\delta = a\sqrt{\frac{2\kappa\pi}{\omega C}}$$

Jet length is between δ and 2δ



Diffusion length

$$\delta = a\sqrt{\frac{2\kappa\pi}{\omega C}}$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{i\omega C}{\kappa\pi a^2} V$$

Boundary conditions:

$$V(z=0) = V_0$$

$$I(z=l_j) \approx 0$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{i\omega C}{\kappa\pi a^2} V$$

Boundary conditions:

$$V(z=0) = V_0$$

$$I(z=l_j) \approx 0$$

$$\delta = a\sqrt{\frac{2\kappa\pi}{\omega C}}$$

$$V(z) = V_0 \frac{\cosh[(1+i)(l_j - z)/\delta]}{\cosh[(1+i)l_j/\delta]}$$

What is the voltage at the tip of the jet?

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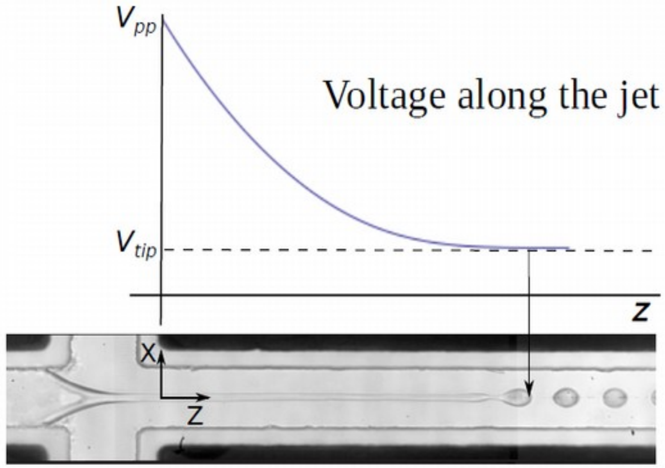

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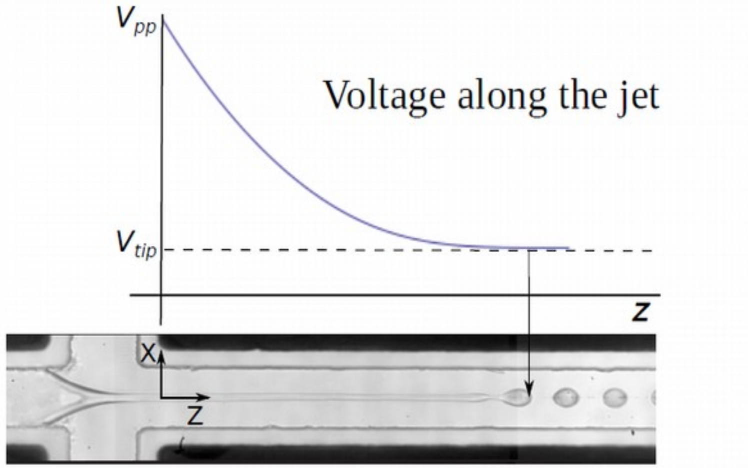

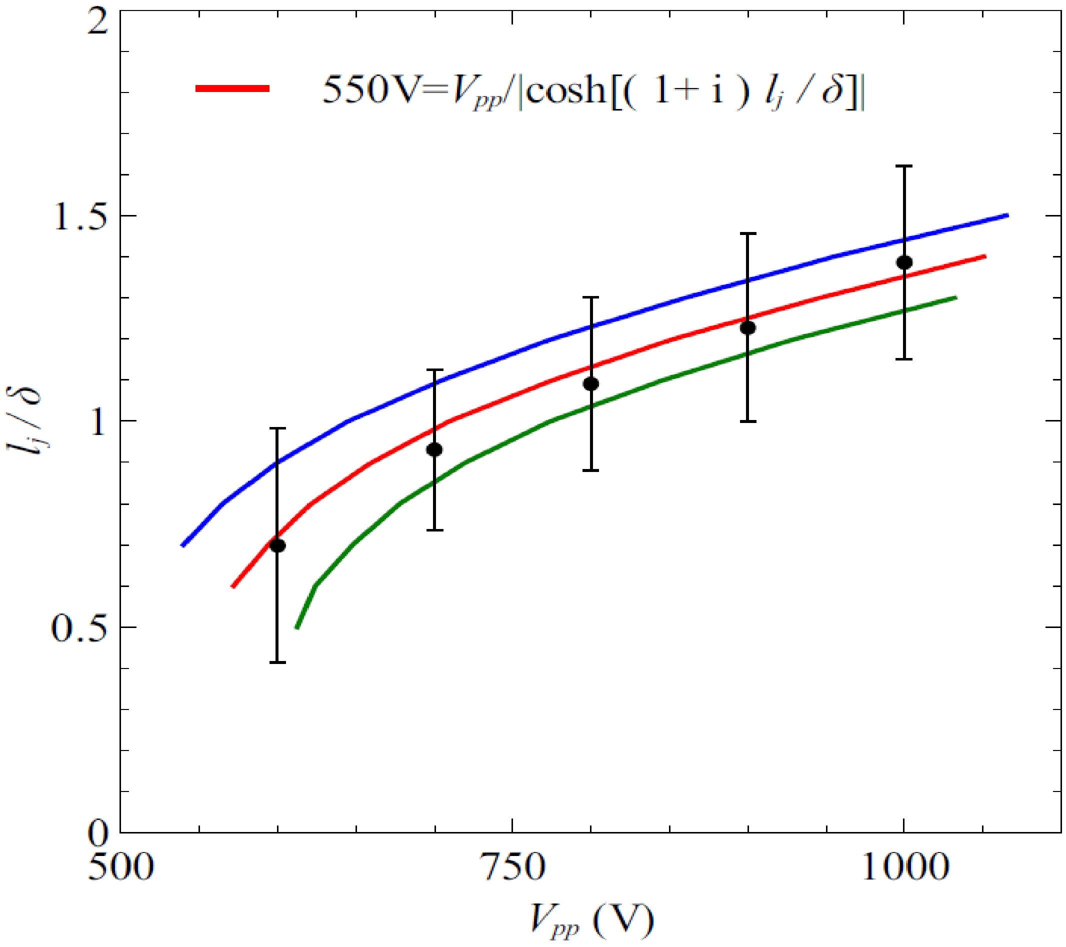

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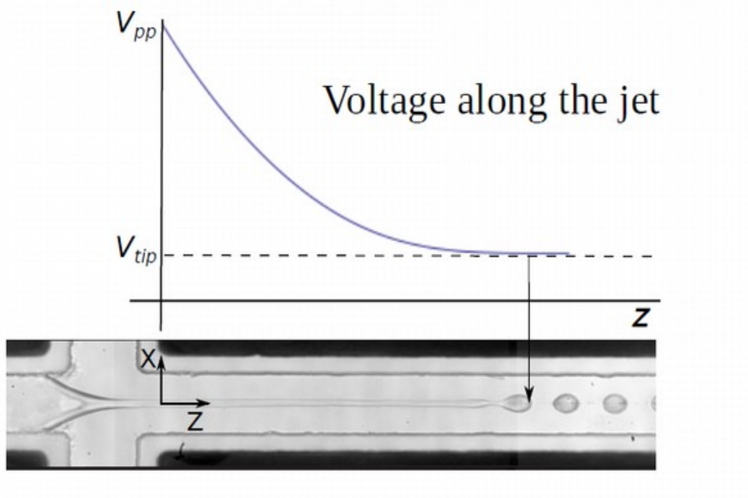
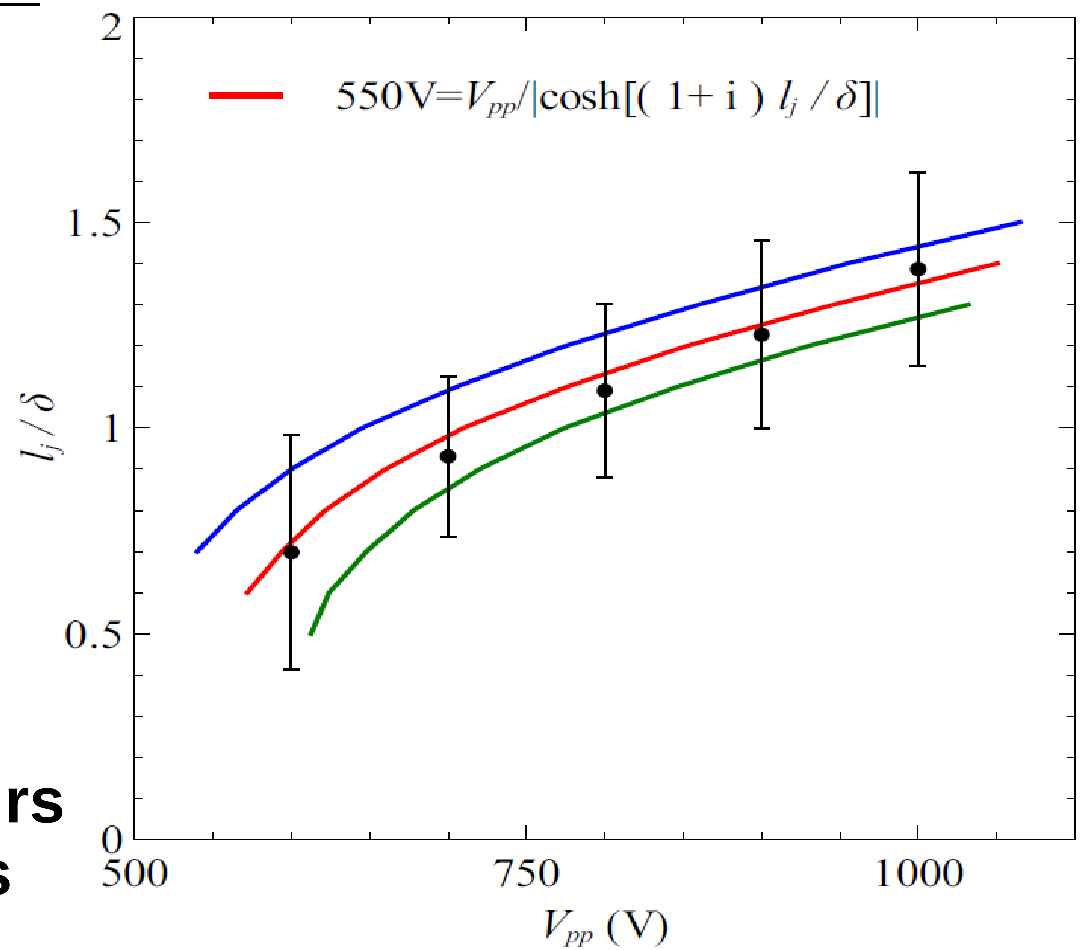


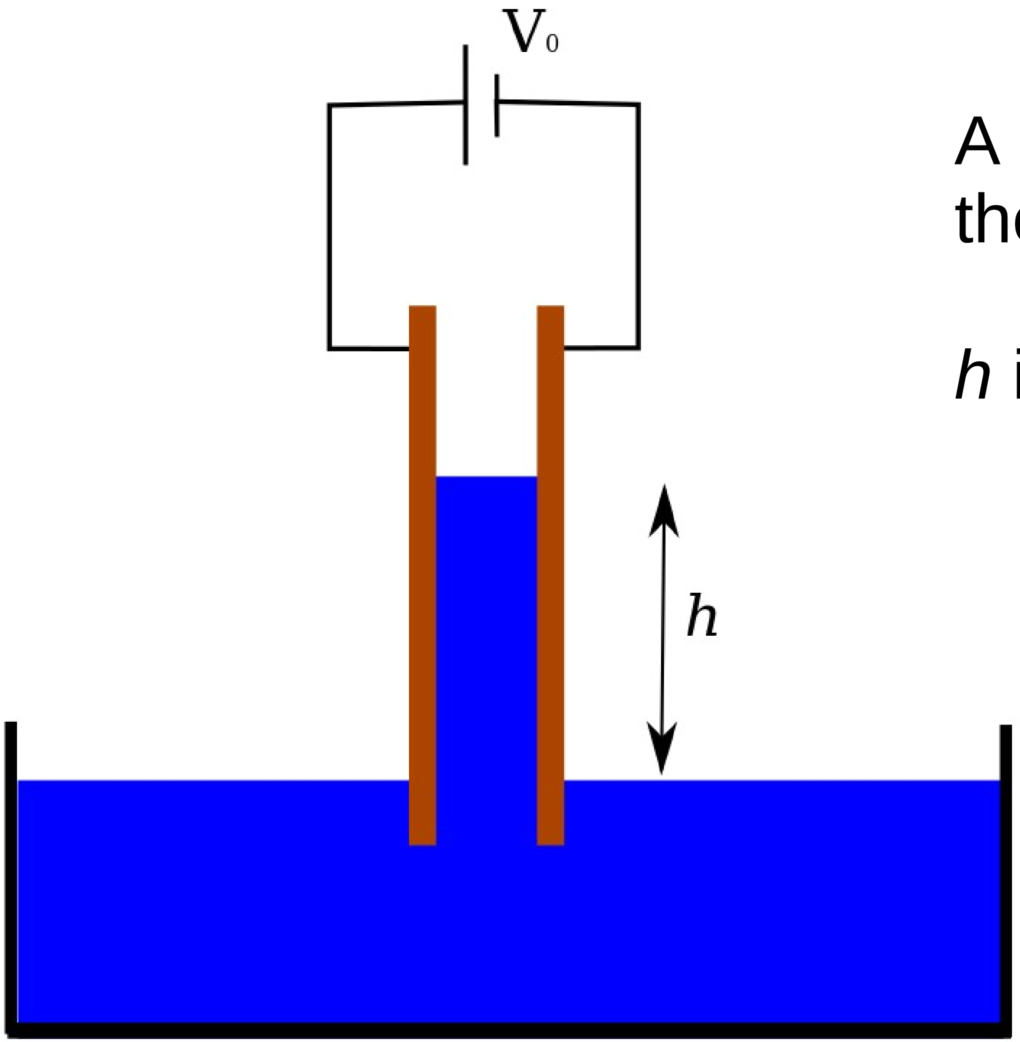


In all cases, jet breakup occurs when the voltage at the tip is around 550 V.

- Why do long jets appear at high voltages?

- What is the electric field along the jet?

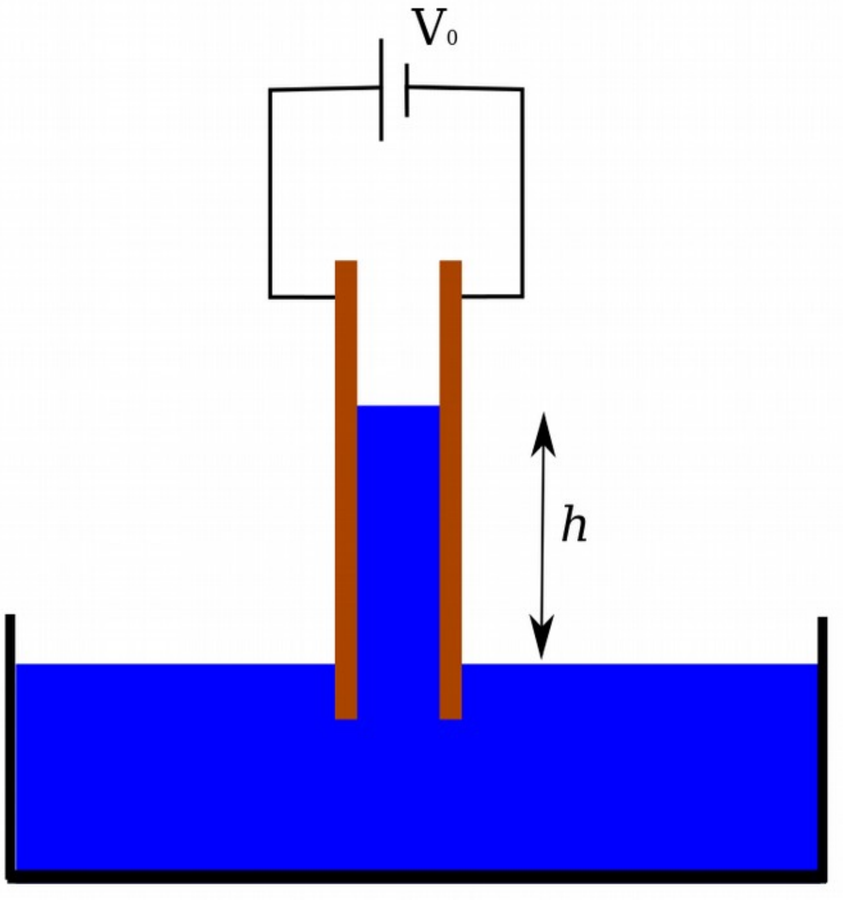
Similarities to Pellat experiment?



A liquid column rises between the electrodes.

h is proportional to applied voltage

Similarities to Pellat experiment?



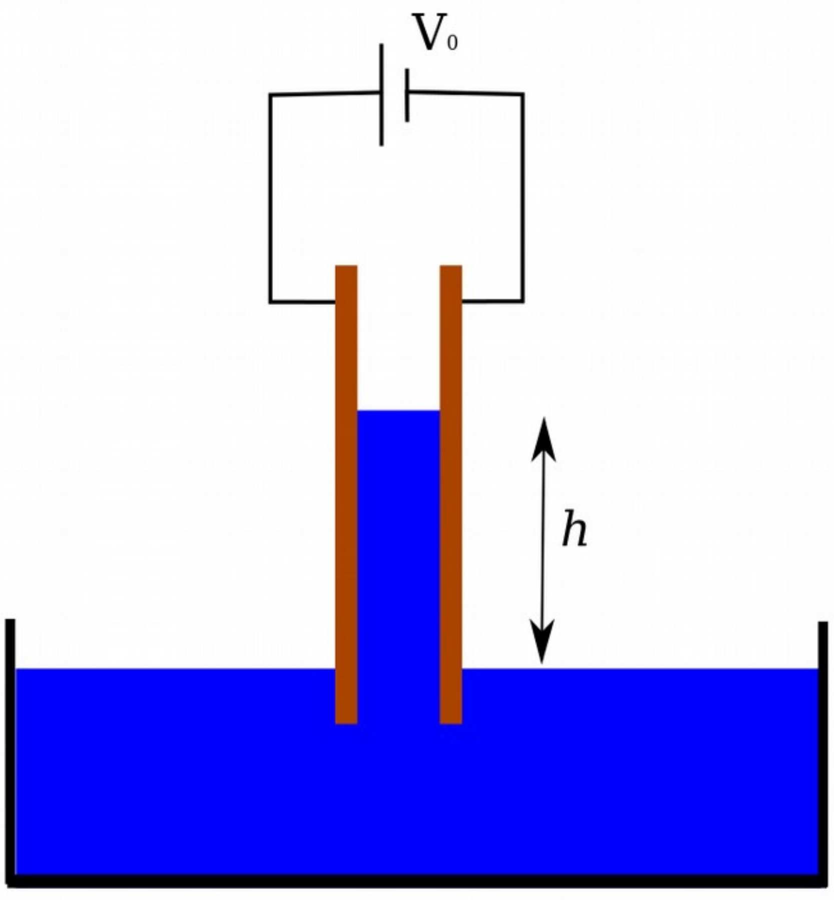
Electrostatic energy

$$U_e = \frac{1}{2}(C_{\text{liquid}} + C_{\text{air}})V_0^2$$

Potential energy (liquid weight)

$$U_g = mgh/2$$

Similarities to Pellat experiment?



Electrostatic energy

$$U_e = \frac{1}{2}(C_{\text{liquid}} + C_{\text{air}})V_0^2$$

Potential energy (liquid weight)

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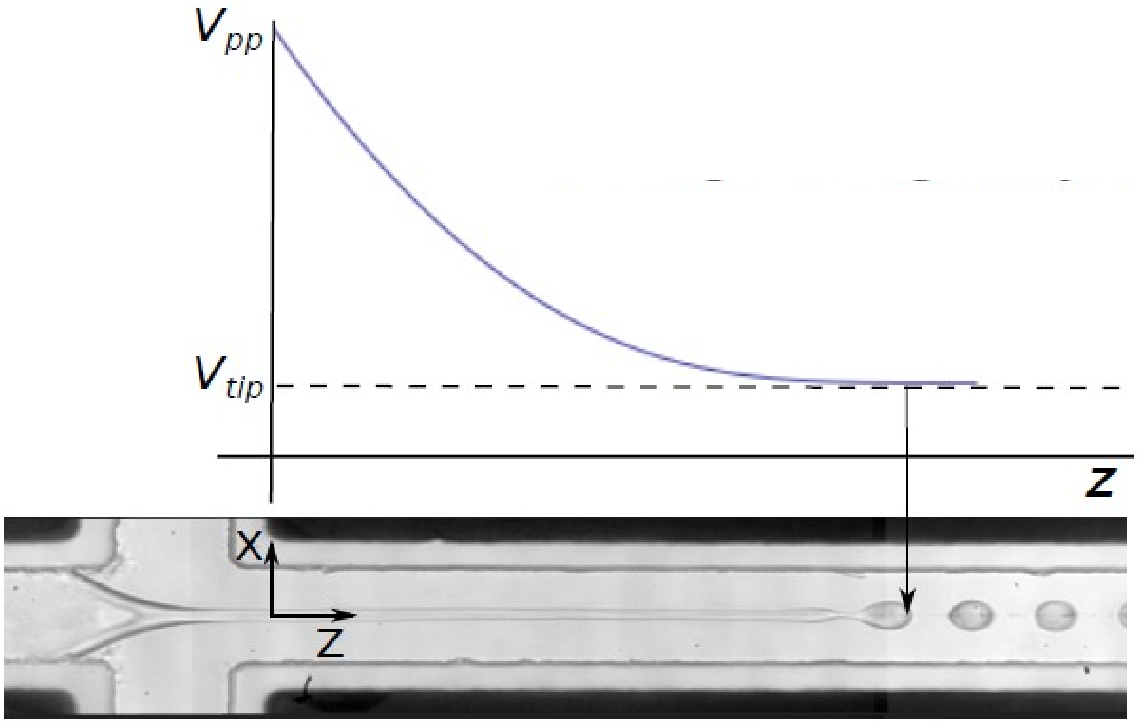
Minimum of free energy determines the equilibrium height:

$$F = U_g - U_e$$

Free energy of the jet



Electrostatic energy per unit length

$$\frac{dU_e}{dz} = \frac{1}{2} C V^2(z)$$

Interfacial energy per unit length

$$\frac{dU_s}{dz} = 2\pi\sigma r$$

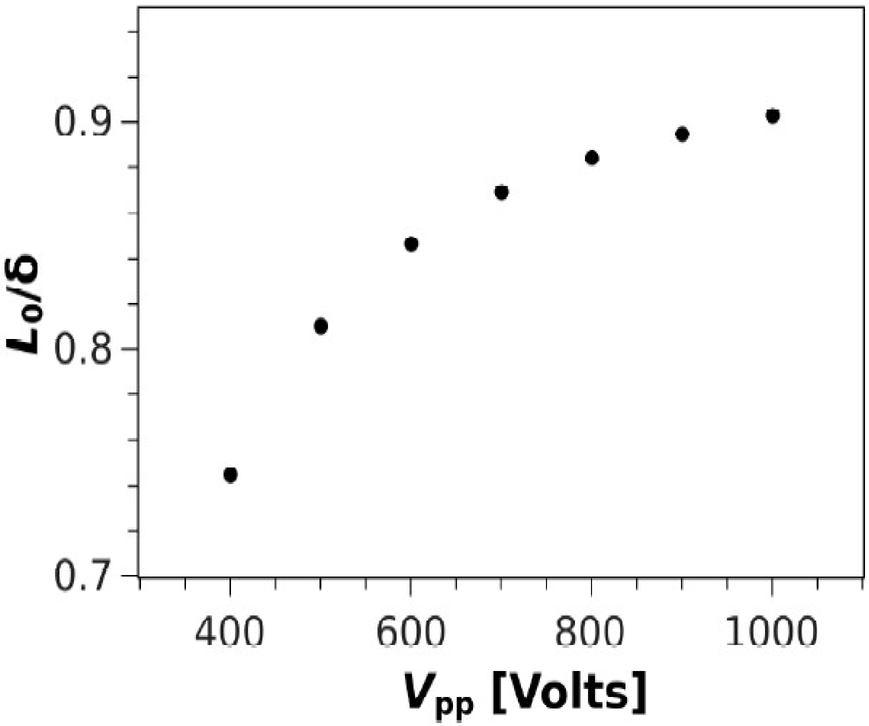
$$F = U_s - U_e,$$

Free energy of the jet

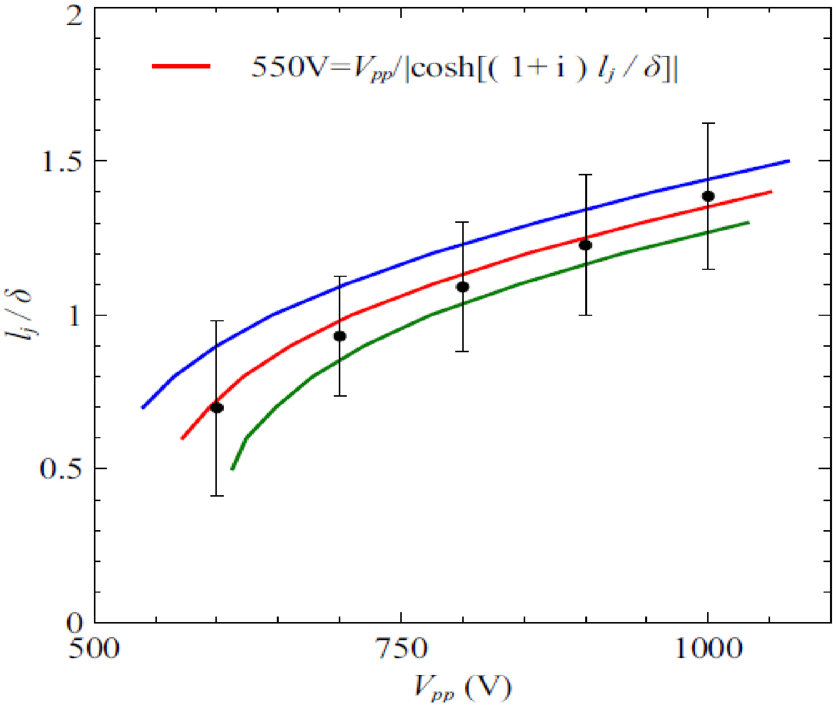
$$F = \int_0^L \left(\frac{dU_s}{dz} - \frac{dU_e}{dz} \right) dz = 2\pi\sigma r L - \frac{1}{2} C \int_0^L V^2(z) dz$$

$$\delta F = 0$$

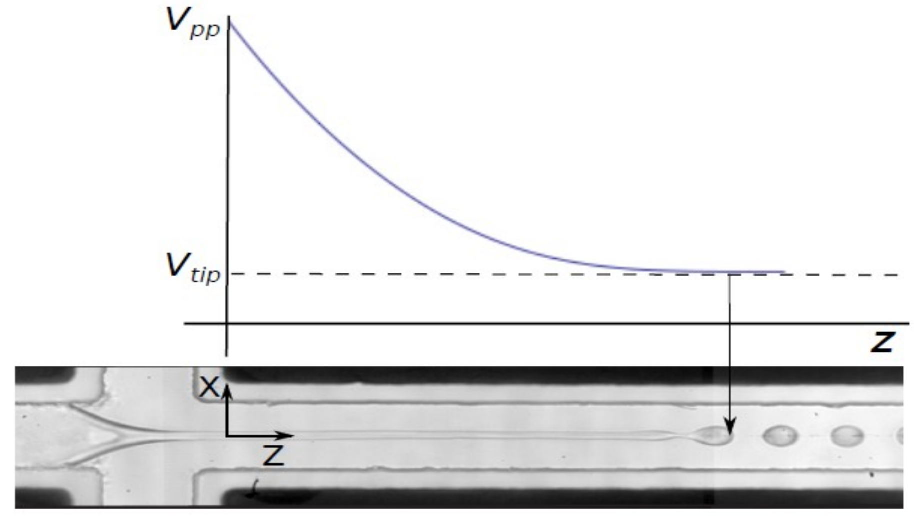
Free energy of the jet



$$\delta F = 0$$

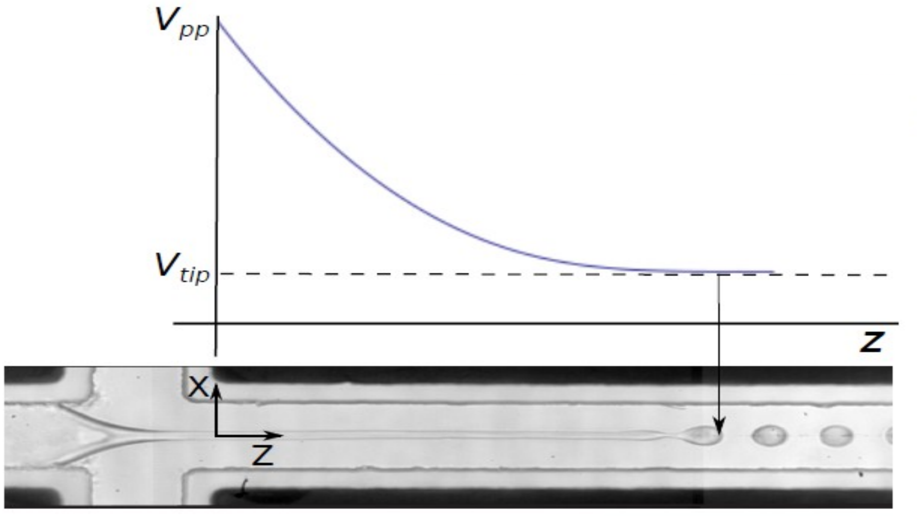


CONCLUSIONS



- **Very slender jets of tunable length are obtained upon application of ac voltages.**

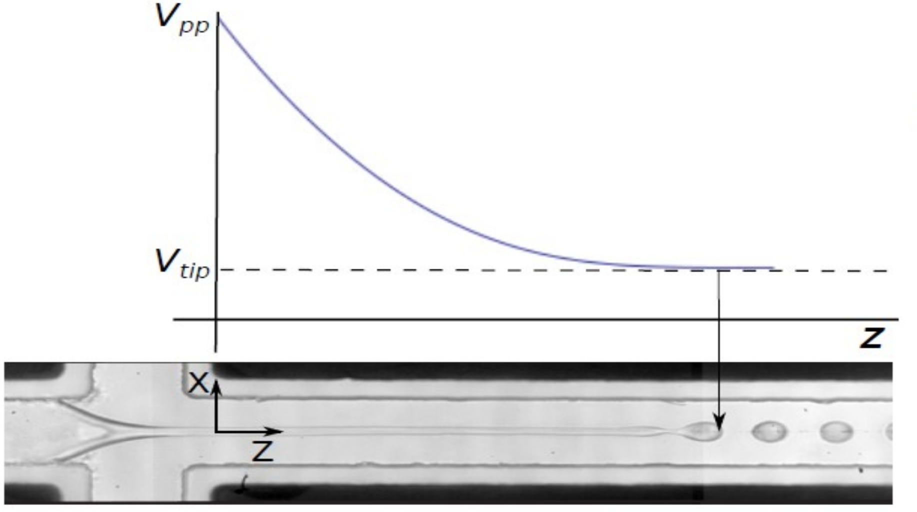
CONCLUSIONS



- **Very slender jets of tunable length are obtained upon application of ac voltages.**

- The **electrical response** of the jet is described with a **discrete element circuit model**. Jet **breakup occurs** when **voltage at the tip is 550 V.**

CONCLUSIONS



- **Very slender jets of tunable length are obtained upon application of ac voltages.**

- The **electrical response** of the jet is described with a **discrete element circuit model**. Jet **breakup occurs** when **voltage at the tip is 550 V.**

- Jet length is given by the competition between **electrical forces** and **interfacial surface tension,** as described via energetic arguments.

Details of length calculation for minimum F

$$F = \int_0^L \left(\frac{dU_s}{dz} - \frac{dU_e}{dz} \right) dz = 2\pi\sigma r L - \frac{1}{2} C \int_0^L V^2(z) dz$$

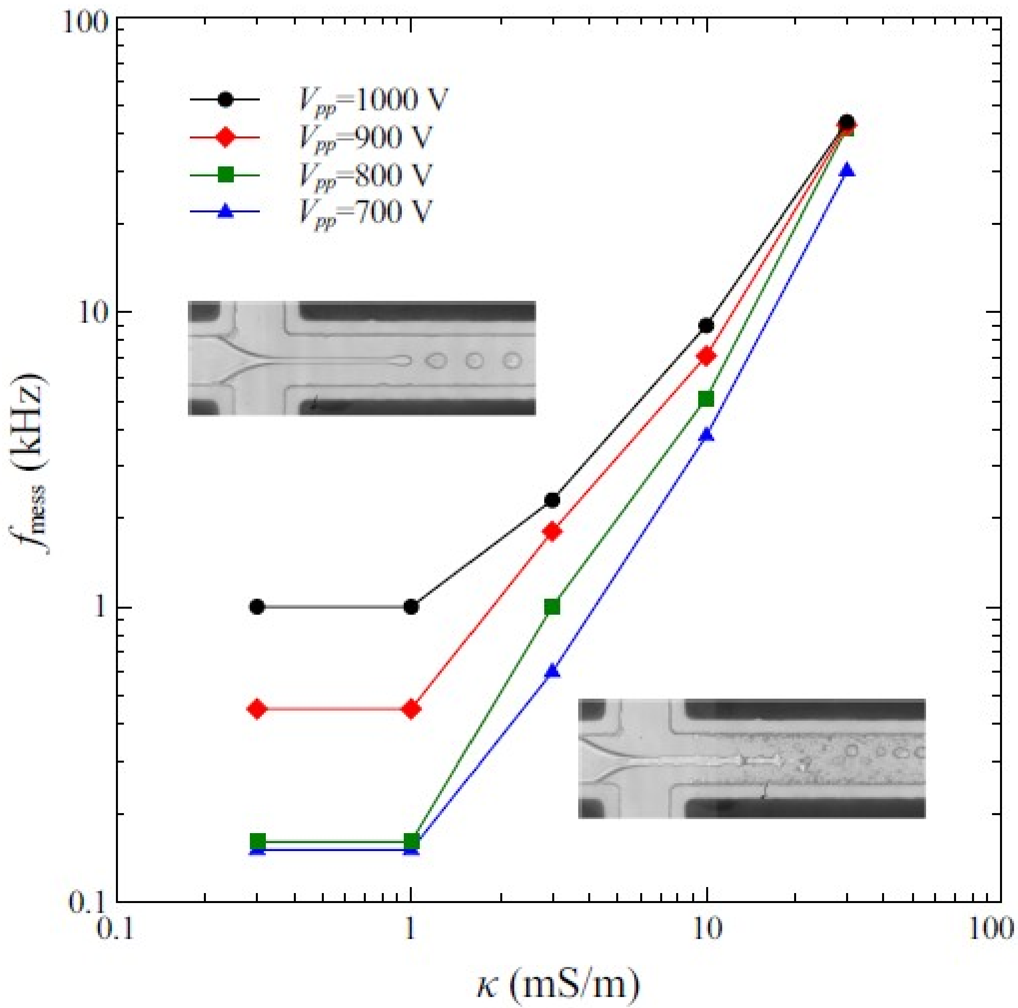
$$\left(\frac{d}{dL} \int_0^L V^2(z) dz \right)_{L=L_0} = \frac{4\pi\sigma r}{C}$$

$$\int_0^L V_{rms}^2(z) dz = \frac{V_0^2}{2} \frac{\delta}{2} \frac{\sinh(2L/\delta) + \sin(2L/\delta)}{\cosh(2L/\delta) + \cos(2L/\delta)}$$

$$f(2L/\delta) = \frac{\eta - 1}{\eta}$$

$$f(x) = \frac{\sinh^2(x) - \sin^2(x)}{(\cosh(x) + \cos(x))^2}$$

$$\eta = CV_0^2 / 8\pi\sigma r.$$



V_{pp}=1000 V
V_{pp}=900 V
V_{pp}=800 V
V_{pp}=700 V
100
10
1
0.1
0.1
1
10
100
f_{mess} (kHz)
κ (mS/m)